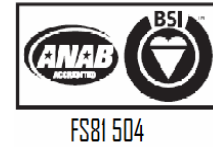




**SHRI ANGALAMMAN COLLEGE OF  
ENGINEERING AND TECHNOLOGY**  
(AN ISO 9001:2008 Certified Institution)  
**SIRUGANOOR, TRICHY – 621 105**



**Department of Computer Science and Engineering**

**QUESTION BANK**

**Subject Code /Title: CS1 303 Theory of Computation Year / Sem: III / V**

**UNIT-I AUTOMATA**

**PART-A(2-MARKS)**

1. List any four ways of theorem proving.
2. Define Alphabets.
3. Write short notes on Strings.
4. What is the need for finite automata?
5. What is finite automata? Give two examples.
6. Define DFA.
7. Explain how DFA process strings.
8. Define transition diagram.
9. Define transition table.
10. Define the language of DFA.
11. Construct a finite automata that accepts  $\{0,1\}^+$ .
12. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings ending in 00.
13. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings with three consecutive 0's.
14. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings with 011 as a substring.
15. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings whose 10<sup>th</sup> symbol from the right end is 1.
16. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings such that each block of 5 consecutive symbol contains at least two 0's.
17. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings that either begins or end(or both) with 01.
18. Give the DFA accepting the language over the alphabet 0,1 that have the set of all strings such that the no of zero's is divisible by 5 and the no of 1's is divisible by 3.
19. Find the language accepted by the DFA given below.
20. Define NFA.
21. Define the language of NFA.
22. Is it true that the language accepted by any NFA is different from the regular language? Justify your Answer.
23. Define -NFA.
24. Define closure.
25. Find the closure for each state from the following automata.

**PART-B**

1 .a)If L is accepted by an NFA with  $\epsilon$ -transition then show that L is accepted by an NFA without  $\epsilon$ -transition.

b)Construct a DFA equivalent to the NFA.

$$M = (\{p, q, r\}, \{0, 1\}, p, \{q, s\})$$

Where  $\delta$  is defined in the following table.

$\delta$	0	1
p	{q,s}	{q}
q	{r}	{q,r}
r	{s}	{p}
s	-	{p}

2. a)Show that the set  $L = \{a^n b^n / n \geq 1\}$  is not a regular. (6)

b)Construct a DFA equivalent to the NFA given below: (10)

	0	1
p	{p,q}	P
q	r	R
r	s	-
s	s	S

3.a)Check whether the language  $L = \{0^n 1^n / n \geq 1\}$  is regular or not? Justify your answer. (6)

b)Let L be a set accepted by a NFA then show that there exists a DFA that accepts L. (10)

4. Define NFA with  $\epsilon$ -transition. Prove that if L is accepted by an NFA with  $\epsilon$ -transition then L is also accepted by a NFA without transition. (16)

5.a)Construct a NDFA accepting all string in  $\{a,b\}^+$  with either two consecutive a's or two consecutive b's. (8)

b)Give the DFA accepting the following language:  
set of all strings beginning with a 1 that when interpreted as a binary integer is a multiple of 5. (8)

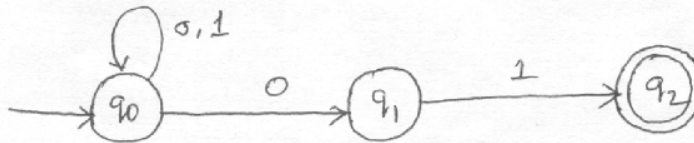
6. Draw the NFA to accept the following languages.

(i) Set of Strings over alphabet  $\{0,1,\dots,9\}$  such that the final digit has appeared before. (8)

(ii) Set of strings of 0's and 1's such that there are two 0's separated by a number of positions that is a multiple of 4. (8)

7.a) Let  $L$  be a set accepted by an NFA. Then prove that there exists a deterministic finite automaton that accepts  $L$ . Is the converse true? Justify your answer. (10)

b) Construct DFA equivalent to the NFA given below: (6)



8.a) Prove that a language  $L$  is accepted by some NFA if and only if  $L$  is accepted by some DFA. (8)

b) Consider the following NFA. Compute the  $\epsilon$ -closure of each state and find its equivalent DFA. (8)

		A	b	C
p	{q}	{p}		
q	{r}		{q}	
*r				{r}

9.a) Prove that a language  $L$  is accepted by some DFA if  $L$  is accepted by some NFA. (8)

b) Convert the following NFA to its equivalent DFA. (8)

	0	1
p	{p,q}	{p}
q	{r}	{r}
r	{s}	
*s	{s}	{s}

10.a) Explain the construction of NFA with transition from any given regular expression. (8)

b) Let  $A = (Q, \Sigma, q_0, \{q_f\})$  be a DFA and suppose that for all  $a$  in  $\Sigma$  we have  $\delta(q_0, a) = q_f$ . Show that if  $x$  is a non empty string in  $L(A)$ , then for all  $k > 0, x^k$  is also in  $L(A)$ . (8)

## UNIT-II REGULAR EXPRESSIONS AND LANGUAGES

### PART-A(2-MARKS)

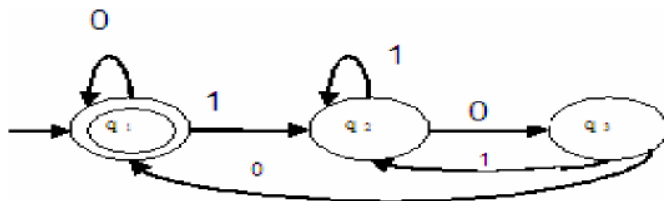
1. Define Regular expression. Give an example.
2. What are the operators of RE.
3. Write short notes on precedence of RE operators.
4. Write Regular Expression for the language that have the set of strings over  $\{a,b,c\}$  containing at least one  $a$  and at least one  $b$ .
5. Write Regular Expression for the language that have the set of all strings of 0's and 1's whose 10<sup>th</sup> symbol from the right end is 1.
6. Write Regular Expression for the language that has the set of all strings of 0's and 1's with at most one pair of consecutive 1's.
7. Write Regular Expression for the language that have the set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.
8. Write Regular Expression for the language that have the set of all strings of 0's and 1's whose no of 0's is divisible by 5.
9. Write Regular Expression for the language that has the set of all strings of 0's and 1's not containing 101 as a substring.
10. Write Regular Expression for the language that have the set of all strings of 0's and 1's such that no prefix has two more 0's than 1's, not two more 1's than 0's.
11. Write Regular Expression for the language that have the set of all strings of 0's and 1's whose no of 0's is divisible by 5 and no of 1's is even.
12. Give English descriptions of the languages of the regular expression  $(1+)(00^*1)^*0^*$ .
13. Give English descriptions of the languages of the regular expression  $(0^*1^*)^*000(0+1)^*$ .
14. Give English descriptions of the languages of the regular expression  $(0+10)^*1^*$ .
15. Convert the following RE to -NFA.  $01^*$ .
16. State the pumping lemma for Regular languages.
17. What are the application of pumping language?
18. State the closure properties of Regular language.
19. Prove that if  $L$  and  $M$  are regular languages then so is  $L \cup M$ .
20. What do you mean by Homomorphism?

21. Suppose  $H$  is the homomorphism from the alphabets  $\{0,1,2\}$  to the alphabets  $\{a,b\}$  defined by  $h(0)=a$   $h(1)=ab$   $h(2)=ba$ . What is  $h(0120)$  and  $h(21120)$ .
22. Suppose  $H$  is the homomorphism from the alphabets  $\{0,1,2\}$  to the alphabets  $\{a,b\}$  defined by  $h(0)=a$   $h(1)=ab$   $h(2)=ba$ . If  $L$  is the language  $L(01^*2)$  what is  $h(L)$ .
23. Let  $R$  be any set of regular languages is  $\cup R_i$  regular? Prove it.
24. Show that the compliment of regular language is also regular.
25. What is meant by equivalent states in DFA.

**PART-B**

1 .a)Construct an NFA equivalent to  $(0+1)^*(00+1 1)$  (16)

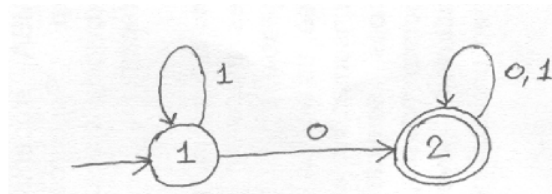
2.a)Construct a Regular expression corresponding to the state diagram given in the following figure. (10)



b)Show that the set  $E=\{0^i 1^j \mid i \geq 1\}$  is not Regular. (6)

3.a)Construct an NFA equivalent to the regular expression  $(0+1)^*(00+1 1)(0+1)^*$ . (8)

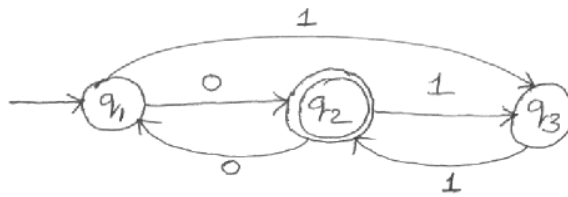
b)Obtain the regular expression that denotes the language accepted by the following DFA. (8)



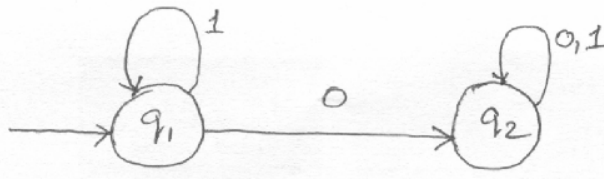
4.a)Construct an NFA equivalent to the regular expression  $((0+1)(0+1)(0+1))^*$  (8)

b)Construct an NFA equivalent to  $10+(0+1 1)0^*1$  (8)

5.a)Obtain the regular expression denoting the language accepted by the following DFA (8)



b) Obtain the regular expression denoting the language accepted by the following DFA by using the formula  $R_{ij, k}$ . (8)



6.a) Show that every set accepted by a DFA is denoted by a regular Expression (8)

b) Construct an NFA equivalent to the following regular expression  $01^*+1$ . (8)

7.a) Define a Regular set using pumping lemma Show that the language  $L=\{0^i / i \text{ is an integer, } i \geq 1\}$  is not regular (8)

b) Construct an NFA equivalent to the regular expression  $10+(0+1)0^*1$  (8)

8.a) Show that the set  $L=\{0^n / n \text{ is an integer, } n \geq 1\}$  is not regular. (6)

b) Construct an NFA equivalent to the following regular expression  $((10)+(0+1)^*0)1$ . (10)

9.a) Prove that if  $L=L(A)$  for some DFA A, then there is a regular expression R such that  $L=L(R)$ . (10)

b) Show that the language  $\{0^p, p \text{ is prime}\}$  is not regular. (6)

10. Find whether the following languages are regular or not.

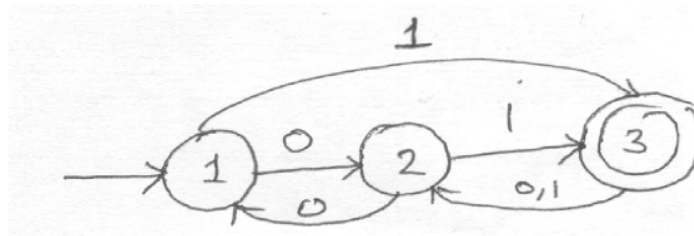
(i)  $L=\{w \{a,b\} | w=w^R\}$ . (4)

(ii)  $L=\{0^n 1^m 2^{n+m}, n, m \geq 1\}$  (4)

(iii)  $L=\{1_k | k=n_2, n \geq 1\}$ . (4)

(iv)  $L_1/L_2 = \{x \mid \text{for some } y \in L_2, xy \in L_1\}$ , where  $L_1$  and  $L_2$  are any two languages and  $L_1/L_2$  is the quotient of  $L_1$  and  $L_2$ . (4)

11 .a) Find the regular expression for the set of all strings denoted by  $R_{213}$  from the deterministic finite automata given below: (8)



b) Verify whether the finite automata  $M_1$  and  $M_2$  given below are equivalent over  $\{a,b\}$ . (8)

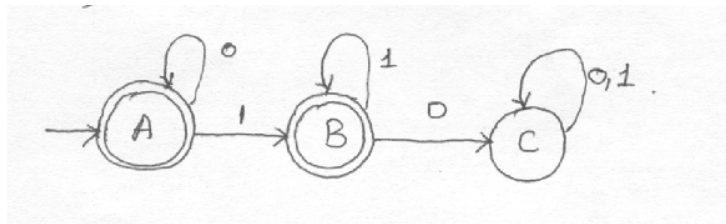
1 2.a) Construct transition diagram of a finite automaton corresponding to the regular expression  $(ab+cb)^*b$ . (8)

b) Construct a minimum state automaton equivalent to a given automaton  $M$  whose transition table is given below.

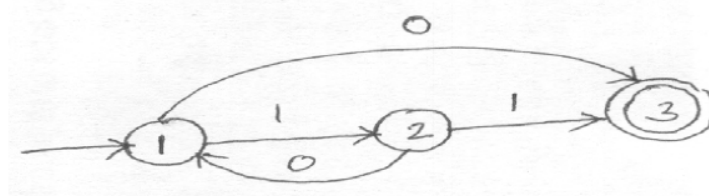
State	Input	
	a	b
q0	q0	q3
q1	q2	q5
q2	q3	q4
q3	q0	q5
q4	q0	q6
q5	q1	q4
q6	q1	q3

(8)

13.a) Find the regular expression corresponding to the finite automaton given below. (8)



b) Find the regular expression for the set of all strings denoted by  $R_{223}$  from the deterministic finite automata given below. (8)



14.a) Find whether the languages  $\{ww, w \text{ is in } (1+0)^*\}$  and  $\{1_k \mid k=n^2, n \geq 1\}$  are regular or not. (8)

b) Show that the regular languages are closed under intersection and reversal. (8)

### UNIT- III CONTEXT FREE GRAMMARS AND LANGUAGES

#### PART-A(2-MARKS)

1. Define CFG.
2. Find  $L(G)$  where  $G = (\{S\}, \{0, 1\}, \{S \rightarrow 0S1, S \rightarrow \epsilon\}, S)$ .
3. Define derivation tree for a CFG (or) Define parse tree.
4. Construct the CFG for generating the language  $L = \{a_n b_n \mid n \geq 1\}$ .
5. Let  $G$  be the grammar  $S \rightarrow aB/bA, A \rightarrow a/aS/bAA, B \rightarrow b/bS/aBB$ . for the string  $aaabbabbba$  find the left most derivation.
6. Let  $G$  be the grammar  $S \rightarrow aB/bA, A \rightarrow a/aS/bAA, B \rightarrow b/bS/aBB$ . obtain parse tree for the string  $aaabbabbba$ .
7. For the grammar  $S \rightarrow aCa, C \rightarrow aCa/b$ . Find  $L(G)$ .
8. Show that  $id+id*id$  can be generated by two distinct leftmost derivation in the grammar  $E \rightarrow E+E \mid E^*E \mid (E) \mid id$ .
9. For the grammar  $S \rightarrow A1 B, A \rightarrow 0A \mid \epsilon, B \rightarrow 0B \mid 1 B \mid \epsilon$ , give leftmost and rightmost derivations for the string  $00101$ .
10. Find the language generated by the CFG  $G = (\{S\}, \{0, 1\}, \{S \rightarrow 0/1, S \rightarrow 0S0/1 S1\}, S)$ .



11. Obtain the derivation tree for the grammar  $G = (\{S, A\}, \{a, b\}, P, S)$  where  $P$  consist of  $S \rightarrow aAS / a$ ,  $A \rightarrow SbA / SS / ba$ .
12. Consider the alphabet  $= \{a, b, (, ), +, *, ., , \}$ . Construct the context free grammar that generates all strings in  $*$  that are regular expression over the alphabet  $\{a, b\}$ .
13. Write the CFG to generate the set  $\{a_m b_n c_p \mid m + n = p \text{ and } p \geq 1\}$ .
14. Construct a derivation tree for the string 0011000 using the grammar  $S \rightarrow A0S \mid 0 \mid SS$ ,  $A \rightarrow S1A \mid 10$ .
15. Give an example for a context free grammar.
16. Let the production of the grammar be  $S \rightarrow 0B \mid 1A$ ,  $A \rightarrow 0 \mid 0S \mid 1AA$ ,  $B \rightarrow 1 \mid 1S \mid 0BB$ . for the string 0110 find the right most derivation.
17. What is the disadvantages of unambiguous parse tree. Give an example.
18. Give an example of PDA.
19. Define the acceptance of a PDA by empty stack. Is it true that the language accepted by a PDA by empty stack or by that of final state are different languages.
20. What is additional feature PDA has when compared with NFA? Is PDA superior over NFA in the sense of language acceptance? Justify your answer.
21. Explain what actions take place in the PDA by the transitions (moves)
 
$$(q, a, Z) = \{(p_1, 1), (p_2, 2), \dots, (p_m, m)\} \text{ and } (q, \epsilon, Z) = \{(p_1, 1), (p_2, 2), \dots, (p_m, m)\}.$$
22. What are the different ways in which a PDA accepts the language? Define them.
23. Is a true that non deterministic PDA is more powerful than that of deterministic PDA? Justify your answer.
24. Explain acceptance of PDA with empty stack.
25. Is it true that deterministic push down automata and non deterministic push down automata are equivalent in the sense of language of acceptances? Justify your answer.
26. Define instantaneous description of a PDA.
27. Give the formal definition of a PDA.

28. Define the languages generated by a PDA using final state of the PDA and empty stack of that PDA.
29. Define the language generated by a PDA using the two methods of accepting a language.
30. Define the language recognized by the PDA using empty stack.

**PART-B**

- 1 .a)Let  $G$  be a CFG and let  $a \Rightarrow w$  in  $G$ . Then show that there is a leftmost derivation of  $w$ . (6)
- b)Let  $G=(V,T,P,S)$  be a Context free Grammar then prove that if  $S \Rightarrow w$  then there is a derivation tree in  $G$  with yield  $w$ . (10)
- 2.a)Let  $G$  be a grammar  $S \rightarrow OS/1A, A \rightarrow O/OS/1AA, B \rightarrow 1/1S/OBB$ .  
For the string 00110101 find its leftmost derivation and derivation tree. (4)
- b)If  $G$  is the grammar  $S \rightarrow Sbs/a$ , Show that  $G$  is ambiguous. (6)
- c)Give a detailed description of ambiguity in Context free grammar.(6)
- 3.a)Show that  $E \rightarrow E+E/E^*E/(E)/id$  is ambiguous. (6)
- b)Construct a Context free grammar  $G$  which accepts  $N(M)$ , where  $M = (\{q_0, q_1\}, \{a, b\}, \{z_0, z\}, q_0, z_0)$  and  $\delta$  is given by  
 $(q_0, b, z_0) = \{(q_0, zz_0)\}$   
 $(q_0, a, z_0) = \{(q_0, z)\}$   
 $(q_0, b, z) = \{(q_0, zz)\}$   
 $(q_0, a, z) = \{(q_1, z)\}$   
 $(q_1, b, z) = \{(q_1, z)\}$   
 $(q_1, a, z_0) = \{(q_0, z_0)\}$  (10)
- 4.a)If  $L$  is Context free language then prove that there exists PDA  $M$  such that  $L = N(M)$ . (10)
- b)Explain different types of acceptance of a PDA.Are they equivalent in sense of language acceptance? Justify your answer. (6)

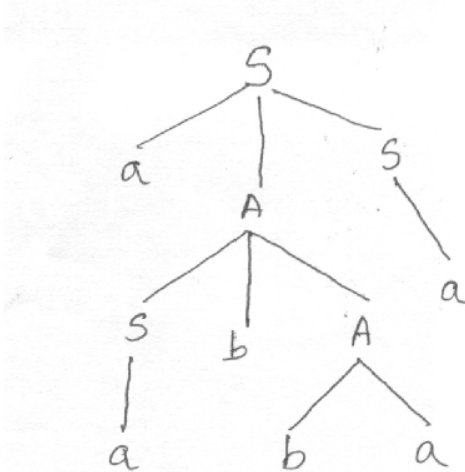
15. Construct a PDA accepting  $\{a^n b^m a^n / m, n \geq 1\}$  by empty stack. Also construct the corresponding context-free grammar accepting the same set. (16)

16.a) Prove that  $L$  is  $L(M_2)$  for some PDA  $M_2$  if and only if  $L$  is  $N(M_1)$  for some PDA  $M_1$ . (10)

b) Define deterministic Push Down Automata DPDA. Is it true that DPDA and PDA are equivalent in the sense of language acceptance is concern? Justify Your answer. (4)

17.a) Construct a equivalent grammar  $G$  in CNF for the grammar  $G_1$  where  $G_1 = (\{S, A, B\}, \{a, b\}, \{S \rightarrow bA/aB, A \rightarrow bAA/aS/a, B \rightarrow aBB/bS/b\}, S)$  (12)

b) Find the left most and right most derivation corresponding to the tree. (4)



18.a) Find the language generated by a grammar  $G = (\{S\}, \{a, b\}, \{S \rightarrow aSb, S \rightarrow ab\}, S)$  (4)

b) Given  $G = (\{S, A\}, \{a, b\}, P, S)$  where  $P = \{S \rightarrow AaS | S | SS, A \rightarrow SbA | ba\}$   $S$ -Start symbol. Find the left most and right most derivation of the string  $w = aabbbaaa$ . Also construct the derivation tree for the string  $w$ . (8)

c) Define a PDA. Give an Example for a language accepted by PDA by empty stack. (4)

19.  $G$  denotes the context-free grammar defined by the following rules.

- $S \rightarrow ASB / ab / SS$
- $A \rightarrow aA / A$
- $B \rightarrow bB / A$

- (i) Give a left most derivation of  $aaabb$  in  $G$ . Draw the associated parse tree.
- (ii) Give a right most derivation of  $aaabb$  in  $G$ . Draw the associated parse tree.

(iii) Show that G is ambiguous. Explain with steps.

(iv) Construct an unambiguous grammar equivalent to G. Explain.  
(4x4=16) 20a) Construct the grammar for the following PDA.

$M = (\{q_0, q_1\}, \{0, 1\}, \{X, Z_0\}, q_0, Z_0)$  and where is given by

$$\begin{array}{ll} (q_0, 0, Z_0) = \{(q_0, XZ_0)\}, & (q_0, 0, X) = \{(q_0, XX)\}, \\ (q_0, 1, X) = \{(q_1, )\}, & (q_1, 1, X) = \{(q_1, )\}, \\ (q_1, X) = \{(q_1, )\}, & (q_1, Z_0) = \{(q_1, )\}. \end{array} \quad (12)$$

b) Prove that if L is  $N(M_1)$  for some PDA  $M_1$  then L is  $L(M_2)$  for some PDA  $M_2$ . (8)

21 .a) Construct a PDA that recognizes the language

$\{a^i b^j c^k \mid i, j, k > 0 \text{ and } i=j \text{ or } i=k\}$ . (8)

b) Discuss about PDA acceptance

- (1) From empty Stack to final state.
- (2) From Final state to Empty Stack. (8)

## UNIT-IV PROPERTIES OF CONTEXTFREE LANGUAGES

### PART-A(2-MARKS)

1. Define multitape Turing Machine.
2. Explain the Basic Turing Machine model and explain in one move. What are the actions take place in TM?
3. Explain how a Turing Machine can be regarded as a computing device to compute integer functions.
4. Describe the non deterministic Turing Machine model. Is it true the non deterministic Turing Machine model's are more powerful than the basic Turing Machines? (In the sense of language Acceptance).
5. Explain the multi tape Turing Machine mode. Is it more power than the basic turing machine? Justify your answer.
6. Using Pumping lemma Show that the language  $L = \{ a^n b^n c^n \mid n \geq 1 \}$  is not a CFL.
7. What is meant by a Turing Machine with two way infinite tape.
8. Define instantaneous description of a Turing Machine.

9. What is the class of language for which the TM has both accepting and rejecting configuration? Can this be called a Context free Language?
10. The binary equivalent of a positive integer is stored in a tape. Write the necessary transition to multiply that integer by 2.
11. What is the role of checking off symbols in a Turing Machine?
12. State Pumping lemma for Context free language.
13. Define a Turing Machine.
14. Mention any two problems which can only be solved by TM.
15. State Pumping lemma and its advantages.
16. What are useless symbols in a grammar.

**PART-B**

- 1 .a) Find a grammar in Chomsky Normal form equivalent to  $S \rightarrow aAD$ ;  $A \rightarrow aB/bAB$ ;  $B \rightarrow b$ ,  $D \rightarrow d$ . (6)
- b) Convert to Greibach Normal Form the grammar  $G = (\{A_1, A_2, A_3\}, \{a, b\}, P, A_1)$  where P consists of the following.  
 $A_1 \rightarrow A_2 A_3$ ,  $A_2 \rightarrow A_3 A_1/b$ ,  $A_3 \rightarrow A_1 A_2/a$ . (10)
- 2.a) Show that the language  $\{0^n 1^{n^2}/n \geq 1\}$  is not a Context free language. (6)
- b) Convert the grammar  $S \rightarrow AB$ ,  $A \rightarrow BS/b$ ,  $B \rightarrow SA/a$  into Greibach Normal Form. (10)
- 3.a) Construct a equivalent grammar G in CNF for the grammar  $G_1$  where  $G_1 = (\{S, A, B\}, \{a, b\}, \{S \rightarrow bA/aB, A \rightarrow bAA/aS/a, B \rightarrow aBB/bS/b\}, S)$  (12)
- b) Obtain the Chomsky Normal Form equivalent to the grammar  $S \rightarrow bA/aB$ ,  $A \rightarrow bAA/aS/a$ ,  $B \rightarrow aBB/bS/b$ . (4)
- 4.a) Begin with the grammar
  - $S \rightarrow 0A0/1 B1/BB$
  - $A \rightarrow C$
  - $B \rightarrow S/A$
  - $C \rightarrow S/$
 and simplify using the safe order
  - Eliminate  $\epsilon$ -Productions
  - Eliminate unit production
  - Eliminate useless symbols
  - Put the (resultant) grammar in Chomsky Normal Form (10)

b) Let  $G=(V,T,P,S)$  be a CFG. Show that if  $S \Rightarrow \epsilon$ , then there is a derivation tree in a grammar  $G$  with yield  $\epsilon$ . (6)

5.a) Let  $G$  be the grammar  $S \rightarrow aS/aSbS/$ . Prove that  $L(G)=\{x \mid \text{each prefix of } x \text{ has at least as many a's as b's}\}$  (6)

b) Explain the Construction of an equivalent grammar in CNF for the grammar  $G=(\{S,A,B\},\{a,b\},P,S)$

where  $P=\{S \rightarrow bA|aB, A \rightarrow bAA|aS|a, B \rightarrow aBB|bS|b\}$  (10)

6.a) Find a Context free grammar with no useless symbol equivalent to

$S \rightarrow AB/CA, B \rightarrow BC/AB$   
 $A \rightarrow a, C \rightarrow aB/b$ . (6)

b) Show that any CFL without  $\epsilon$  can be generated by an equivalent grammar in Chomsky Normal Form. (10)

7.a) Convert the following CFG to CNF

$S \rightarrow ASA|aB$   
 $A \rightarrow B|S$   
 $B \rightarrow b|$  (12)

b) Explain about Greibach Normal Form. (4)

8.a) Is  $L=\{a_n b_n c_n \mid n \geq 1\}$  a context free language? Justify Your answer. (8)

b) Prove that for every context free language  $L$  without  $\epsilon$  there exists an equivalent grammar in Greibach Normal Form. (8)

9. State and Prove pumping lemma for Context free languages. (16)

10.a) State Pumping Lemma for context free language. Show that  $\{0^n 1^n 2^n \mid n \geq 1\}$  is not a Context free language. (6)

b) State Pumping lemma for context free language show that language  $\{a^i b^j c^k \mid i \geq 1, \text{ and } j \geq 1\}$  is not context-free. (6)

11 .a) Design a Turing Machine  $M$  to implement the function "multiplication" using the subroutine 'copy'. (12)

- b) Explain how a Turing Machine with the multiple tracks of the tape can be used to determine the given number is prime or not. (4)
- 12.a) Design a Turing Machine to compute  $f(m+n)=m+n, \forall m,n \geq 0$  and simulate their action on the input 0100. (10)
- b) Describe the following Turing machine and their working. Are they more powerful than the Basic Turing Machine?  
 Multi-tape Turing Machine  
 Multi-Dimensional Turing Machine  
 (3) Non-Deterministic Turing Machine. (6)
- 13.a) Define Turing machine for computing  $f(m,n)=m-n$  ( proper subtraction). (10)
- b) Explain how the multiple tracks in a Turing Machine can be used for testing given positive integer is a prime or not. (6)
- 14.a) Explain in detail: " The Turing Machine as a Computer of integer functions". (8)
- b) Design a Turing Machine to accept the language  $L=\{0^n 1^n/n \geq 1\}$  (8)
- 15.a) What is the role of checking off symbols in a Turing Machine? (4)
- b) Construct a Turing Machine that recognizes the language  $\{wcw/w \text{ in } \{a+b\}^+\}$  (12)
16. Prove that the language L is recognized by a Turing Machine with a two way infinite tape if and only if it is recognized by a Turing Machine with a one way infinite tape. (16)
17. For each of the following Context free languages L, find the smallest pumping length that will satisfy the statement of the Context free pumping lemma. In each case, Your answer should include a number (the minimum pumping length), a detailed explanation of why that the number is indeed a valid pumping length for the given language L, and a detailed explanation of why no smaller number qualifies as a valid pumping length for that particular language L.
- (i)  $L=\{a^n b^n | n \geq 0\}$  (6)
- (ii)  $L=\{w \text{ in } \{a,b\}^* | w \text{ has the same number of a's and b's}\}$  (6)
- (iii)  $L=\{w \text{ in } \{a,b\}^* | w \text{ has twice as many a's as b's.}\}$  (4)
18. Design a Turing Machine M that decides  $A=\{0^k / n > 0 \text{ and } k=2^n\}$  the language consisting of all strings of 0's whose length is a power of 2. (16)

19.a) Give a High level implementation description with a neat sketch of a Turing Machine  $M$  that performs the following computation.  $M$  on input  $w$ : writes a copy of  $w$  on the tape immediately after  $w$ , leaving the string  $w\#w$  on the tape. Assume that the input string initially appears at the left most end of the tape and that the input alphabet does not contain the blank character ' '. The end of the input string is therefore determined by the location of the first blank cell on the input tape. The symbol  $\#$  is assumed to be in the tape alphabet, and the input alphabet is  $\{a,b\}$ .

(12)

b) Demonstrate the working of your TM with an example. (4)

20.a) Show that the language  $\{0^n 1^n 2^n / n \geq 1\}$  is not context free.

(8)

b) Show that the context free languages are closed under union operation but not under intersection. (8)

## UNIT-V

## UNDECIDABILITY

### PART-A (2-MARKS)

1. When a recursively enumerable language is said to be recursive.
2. Is it true that the language accepted by a non deterministic Turing Machine is different from recursively enumerable language?
3. When we say a problem is decidable? Give an example of undecidable problem?
4. Give two properties of recursively enumerable sets which are undecidable.
5. Is it true that complement of a recursive language is recursive? Justify your answer.
6. When a language is said to be recursive or recursively enumerable?
7. When a language is said to be recursive? Is it true that every regular set is not recursive?
8. When a problem is said to be decidable or undecidable? Give an example of an undecidable.
9. What do you mean by universal Turing Machine?
10. When a problem is said to be undecidable? Give an example of an decidable problem.
11. Show that the union of recursive language is recursive.



12. Show that the union of two recursively enumerable languages is recursively enumerable.
13. What is undecidability problem?
14. Show that the following problem is undecidable. "Given two CFG's  $G_1$  and  $G_2$ , is  $L(G_1) \cap L(G_2) = \emptyset$ ?"
15. Define  $L_d$ .
16. Define recursively enumerable language.
17. Give an example for a non recursively enumerable language.
19. Differentiate between recursive and recursively enumerable languages.
20. Mention any two undecidability properties for recursively enumerable language.
21. Define Diagonal languages.
22. Give an example for an undecidable problem.

**PART-B**

1. a) Show that union of recursive languages is recursive. (4)
- b) Define the language  $L_d$  and show that  $L_d$  is not recursively enumerable language. (8)
- c) Explain the Halting problem. Is it decidable or undecidable problem (4)
  
2. Define Universal language  $L_u$ . Show that  $L_u$  is recursively enumerable but not recursive. (16)
  
3. a) Obtain the code for the TM  
 $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, q_1, B, \{q_2\})$   
 With the moves  
 $(q_1, 1) = (q_3, 0, R)$   
 $(q_3, 0) = (q_1, 1, R)$   
 $(q_3, 1) = (q_2, 0, R)$   
 $(q_3, B) = (q_3, 1, L)$   
 $(q_3, B) = (q_3, 1, L)$  (4)
- b) Show that  $L_n$  is recursively enumerable. (12)

- 4.a) Define  $L_d$  and show that  $L_d$  is not recursively enumerable. (12)
- b) Whether the problem of determining given recursively enumerable Language is empty or not? Is decidable? Justify your answer. (4)
5. Define the language  $L_u$ . Check whether  $L_u$  is recursively enumerable? or  $L_u$  is recursive? Justify your answer. (16)
- 6.a) Show that the language  $L_d$  is neither recursive nor recursively enumerable. (12)
- b) Describe how a Turing Machine can be encoded with 0 and 1 and give an example. (4)
- 7.a) Show that any non trivial property  $J$  of the recursively enumerable languages is undecidable. (8)
- b) Show that if  $L$  and  $L$  are recursively enumerable then  $L$  and  $L$  recursive. (8)
8. Define the universal language and show that it is recursively enumerable but not recursive. (16)
9. Prove that the universal language  $L_u$  is recursively enumerable. (16)
10. State and Prove Rice's Theorem for recursive index sets. (16)
- 11 .a) Show that the following language is not decidable.  
 $L = \{ \langle M \rangle \mid M \text{ is a TM that accepts the string } aaab \}$ . (8)
- b) Discuss the properties of Recursive and Recursive enumerable languages. (8)
- 12.a) Define Post correspondence problem with an example. (8)
- b) Prove that the function  $f(n) = 2^n$  does not grow at a polynomial rate, in other words, it does not satisfy  $f(n) = O(n^p)$  for any finite exponent  $p$ . (8)
- 13.a) Define the language  $L_d$ . Show that  $L_d$  is neither recursive nor recursively enumerable. (12)
- b) Show that if a language  $L$  and its complement  $L$  are both recursively enumerable then  $L$  is recursive. (4)

14.a) What are the features of a Universal Turing Machine? (4)

b) Show that "If a language  $L$  and its complement  $\bar{L}$  are both recursively enumerable, then both languages are recursive". (6)

c) Show that halting problem of Turing Machine is undecidable. (6)

15.a) Does PCP with two lists  $x=(b, b, ab^3, ba)$  and  $y=(b^3, ba, a)$  have a solution?. (6)

b) Show that the characteristic function of the set of all even numbers is recursive. (6)

c) Let  $\Sigma = \{0, 1\}$ . Let  $A$  and  $B$  be the lists of three strings each, defined as

List A

List B

$i$	$W_i$	$X_i$
1	1	111
2	10111	10
3	10	0

Does this PCP have a solution? (4)

16.a) Show that it is undecidable for arbitrary CFG's  $G_1$  and  $G_2$  whether  $L(G_1) \cap L(G_2)$  is a CFL. (8)

b) Show that "finding whether the given CFG is ambiguous or not" is undecidable by reduction technique. (8)

17. Find whether the following languages are recursive or recursively enumerable.

(i) Union of two recursive languages. (4)

(ii) Union of two recursively enumerable languages. (4)

(iii)  $L$  if  $L$  and complement of  $L$  are recursively enumerable. (4)

(iv)  $L_u$  (4)

18. Consider the Turing Machine  $M$  and  $w=01$ , where  $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, q_1, B, \{q_3\})$  and  $\delta$  is given by

$q_i$	$(q_i, 0)$	$(q_i, 1)$	$(q_i, B)$
Q1	$(q_2, 1, R)$	$(q_2, {}^0L)$	$(q_2, 1, L)$
q2	$(q_3, {}^0L)$	$(q_1, {}^0R)$	$(q_2, 0, R)$
q3	--	--	--

Reduce the above problem to Post's correspondence Problem and find whether that PCP has a solution or not. (16)

19. Explain the Post's Correspondence Problem with an example (16)

20. Find the languages obtained from the following operations:

- (i) Union of two recursive languages. (6)
- (ii) Union of two recursively enumerable languages (6)
- (iii) L if L and complement of L are recursively enumerable (4)

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